

MATH 290 - QUIZ #1

Name: Key

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(3 points) 1. Solve the following differential equation: $\frac{dy}{dx} = x\sqrt{1-y^2}$

$$\frac{dy}{\sqrt{1-y^2}} = x dx$$

$$\sin^{-1} y = \frac{x^2}{2} + C$$

(4 points) 2. Solve the following differential equation: $\frac{dy}{dx} + 4xy = x^3 e^{x^2}$

$$\mu(x) = e^{\int 4x dx} = e^{2x^2}$$

$$e^{2x^2} y = \int x^3 e^{x^2} \cdot e^{2x^2} dx = \int x^3 e^{3x^2} dx$$

$$u = x^2 \quad dw = x e^{3x^2} dx$$

$$du = 2x dx \quad w = 3x^2$$

$$dw = 6x dx$$

$$\frac{1}{6} dw = x dx$$

$$v = \frac{1}{6} e^{3x^2}$$

$$\frac{1}{6} x^2 e^{3x^2} - \frac{1}{3} \int x e^{3x^2} dx$$

$$e^{2x^2} y = \frac{1}{6} x^2 e^{3x^2} - \frac{1}{18} e^{3x^2} + C$$

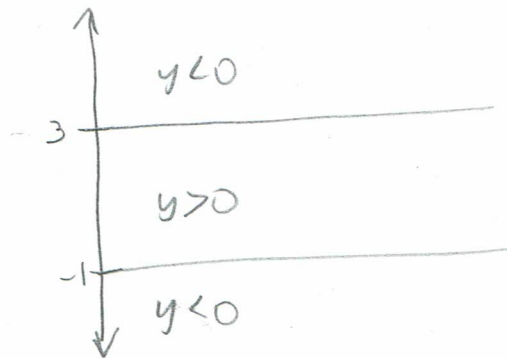
3. Given the differential equation $\frac{dy}{dx} = 3 + 2y - y^2$.

(1 point) a. Determine all equilibrium solutions.

$$-(y^2 - 2y - 3) = -(y - 3)(y + 1)$$

$$y = 3, y = -1$$

(3 points) b. Determine the regions when the solutions are increasing or decreasing.

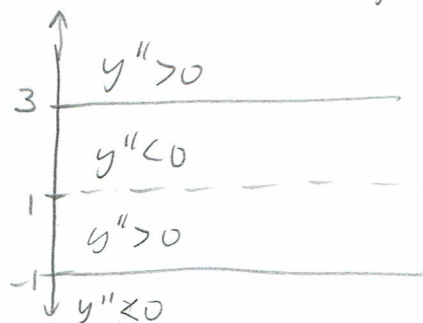


increasing on $(-1, 3)$

decreasing on $(-\infty, -1) \cup (3, \infty)$

(3 points) c. Determine the regions when the solutions are concave up or concave down.

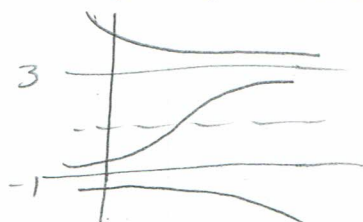
$$\begin{aligned} y'' &= (2 - 2y)y' = 2(1 - y)(-y - 3)(y + 1) \\ &= -2(1 - y)(y - 3)(y + 1) \end{aligned}$$



concave up $(-1, 1) \cup (3, \infty)$

concave down $(-\infty, -1) \cup (1, 3)$

(2 points) d. Classify the equilibrium solutions as stable or unstable.



$y = -1$ unstable

$y = 3$ stable.