MATH 290 – QUIZ #2 *Name:* KEY

Directions: This is a take home quiz. This quiz is due at the beginning of class on Monday, March 25, 2019. Please show all work for maximum credit. This quiz is worth 10 points. Good luck!

(4 points) 1. Solve the following differential equation: $\left(y^2+xy\right)dx + x^2 dy = 0$

$M(tx,ty) = t^2y^2 + tx\,ty = t^2(y^2+xy) = t^2 M(x,y)$

$N(tx,ty) = t^2x^2 = t^2 N(x,y)$

$\therefore$ homogeneous of degree 2.

Let $u = \frac{y}{x}$, $y = ux$

So, $dy = u\,dx + x\,du$

$\left((ux)^2 + x(ux)\right)dx + x^2(u\,dx + x\,du) = 0$

$(u^2x^2 + ux^2)\,dx + x^2(u\,dx + x\,du) = 0$

$u^2x^2\,dx + 2ux^2\,dx + x^3\,du = 0$

$x^2(u^2+2u)\,dx = -x^3\,du$

$-\frac{1}{x}\,dx = \frac{1}{u^2+2u}\,du$

$-\frac{1}{x}\,dx = \frac{1}{u(u+2)}\,du$

$-\frac{1}{x}\,dx = \left(\frac{\frac{1}{2}}{u} - \frac{\frac{1}{2}}{u+2}\right)du$

$-\ln|x| + C = \frac{1}{2}\ln|u| - \frac{1}{2}\ln|u+2|$

$-\ln|x| + C = \frac{1}{2}\ln\left|\frac{y}{x}\right| - \frac{1}{2}\ln\left|\frac{y}{x}+2\right|$

$\frac{1}{u(u+2)} = \frac{A}{u} + \frac{B}{u+2}$

$1 = A(u+2) + Bu$

Let $u = 0$ $A = 1/2$

Let $u = -2$ $B = -1/2$

(4 points) *2.* Solve the following differential equation: $\left(3x^2y+e^y\right)dx+\left(x^3+xe^y-2y\right)dy=0$

$M_y = 3x^2+e^y \qquad N_x = 3x^2+e^y \quad \therefore$ exact

$f(x,y) = \int M\,dx = \int (3x^2y+e^y)\,dx = x^3y+xe^y+g(y)$

$\frac{\partial f}{\partial y} = x^3+xe^y+g'(y) = N(x,y)$

$x^3+xe^y+g'(y) = x^3+xe^y-2y$

$g'(y) = -2y$

$g(y) = -y^2+C_1$

$f(x,y) = x^3+xe^y-y^2+C_1$

Solution: $x^3+xe^y-y^2 = C$

(2 points) *3.* Determine the integrating factor necessary to change the following differential equation into an exact equation. Do not solve the differential equation.

$$\left(x^2+y^2-5\right)dx-\left(y+xy\right)dy=0$$

$M_y = 2y \qquad N_x = -y$

$\frac{M_y-N_x}{N} = \frac{3y}{-(y+xy)} = -\frac{3y}{y(1+x)} = -\frac{3}{1+x} = f(x)$

$\mu(x) = e^{-\int \frac{3}{1+x}dx} = e^{-3\ln|1+x|} = (1+x)^{-3}$