MATH 290 – QUIZ #2 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(6 points) 1. A tank contains 40 L of a solution into which 10 g of salt is dissolved. A solution containing 2 g/L of salt flows into the tank at a rate of 5 L/min, and the well-stirred mixture flows out at a rate of 2 L/min. What is the concentration of salt in the tank after 30 minutes?

$V(0) = 40L \qquad A(0) = 10g$

$\frac{dV}{dt} = 5 - 2 = 3$

$dV = 3dt$

$V = 3t + C_1$

$V(0) = 40$

$C_1 = 40$

$V = 3t + 40$

$\frac{dA}{dt} = 2\frac{g}{L} \cdot 5\frac{L}{min} - \frac{A}{V}\frac{g}{L} - 2\frac{L}{min}$

$\frac{dA}{dt} + \frac{2}{3t+40}A = 10$

$\mu(t) = e^{\int \frac{2}{3t+40}dt}$

$= e^{\frac{2}{3}\ln|3t+40|} = (3t+40)^{2/3}$

$\frac{d}{dt}\left[(3t+40)^{2/3}A\right] = 10(3t+40)^{2/3}$

$(3t+40)^{2/3}A = 10\int(3t+40)^{2/3}dt$

$(3t+40)^{2/3}A = 12\,(3t+40)^{5/3} + C_2$

$A = 2(3t+40) + C_2(3t+40)^{-2/3}$

$10 = 80 + C_2(40)^{-2/3}$

$-70 = C_2(40)^{-2/3}$

$C_2 = -70(40)^{2/3}$

$A = 2(3t+40) - 70(40)^{2/3}(3t+40)^{-2/3}$

$A(30) = 2(3(30)+40) - 70(40)^{2/3}(3(30)+40)^{-2/3}$

$A \approx 226.0961962\,g$

$V(30) = 3(30)+40 = 130\,L$

Concentration $\approx 1.75\,g/L$

(5 points) 2. Consider a 20-volt electromotive force that is applied to an *RL*-series circuit in which the resistance is 4 ohms and the inductance is 0.1 henry. Find the current $i(t)$ on the capacitor if $i(0)=0$.

$$\frac{1}{10}\frac{di}{dt}+4i=20$$

$$\frac{di}{dt}+40i=200$$

$$\mu(t)=e^{\int 40\,dt}=e^{40t}$$

$$\frac{d}{dt}\left[e^{40t}i\right]=200e^{40t}$$

$$e^{40t}i=200\int e^{40t}\,dt$$

$$e^{40t}i=5e^{40t}+C$$

$$i=5+Ce^{-40t}$$

$$0=5+C$$

$$C=-5$$

$$i=5-5e^{-40t}$$

(5 points) 3. A small metal bar, whose initial temperature was 30°C, is dropped into a large container of water with a temperature of 90°C. How long will it take the bar to reach 70°C if it is known that its temperature increases 2° in one second?

$$T=T_m+Ce^{kt}$$

$$T=90+Ce^{kt}$$

$$T(0)=30$$

$$30=90+C$$

$$C=-60$$

$$T=90-60e^{kt}$$

$$T(1)=32$$

$$32=90-60e^{k}$$

$$-58=-60e^{k}$$

$$\frac{58}{60}=e^{k}$$

$$\ln 29/30=k$$

$$T=90-60e^{(\ln 29/30)t}$$

$$70=90-60e^{(\ln 29/30)t}$$

$$-20=-60e^{(\ln 29/30)t}$$

$$\frac{1}{3}=e^{(\ln 29/30)t}$$

$$\ln 1/3=(\ln 29/30)t$$

$$t=\frac{\ln 1/3}{\ln 29/30}$$

$$t=32.4 \text{ seconds.}$$