MATH 290 – QUIZ #2 *Name:* KEY

Directions: This is a take home quiz. This quiz is due at the beginning of class on Monday, January 28, 2019. Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(4 points) 1. A tank whose volume is 200 L is initially half full of a solution that contains 100 g of chemical. A solution containing 0.5 g/L of the same chemical flows into the tank at a rate of 6 L/min, and the well-stirred mixture flows out at a rate of 4 L/min. Determine the concentration of chemical in the tank just before the solution overflows. $V(0)=100L$ $A(0)=100g$

$\frac{dV}{dt}=6-4=2$

$dV=2\,dt$

$\int dV=\int 2\,dt$

$V=2t+C_1$

$100=2(0)+C_1$

$C_1=100$

$V=2t+100$

$V=2t+100$

Tank overflows when $V=200L$

$200=2t+100$

$100=2t$

$t=50$ min

$\frac{dA}{dt}=C_{in}r_{in}-C_{out}r_{out}$

$\frac{dA}{dt}=(0.5\frac{g}{L})(6\frac{L}{min})-(\frac{A}{V}\frac{g}{L})(4\frac{L}{min})$

$\frac{dA}{dt}=3-\frac{A}{2t+100}\cdot 4$

$\frac{dA}{dt}+\frac{2}{t+50}A=3$

$\mu(t)=e^{\int\frac{2}{t+50}dt}=e^{2\ln|t+50|}=(t+50)^2$

$\frac{d}{dt}[(t+50)^2A]=3(t+50)^2$

$(t+50)^2A=3\int(t+50)^2dt$

$(t+50)^2A=(t+50)^3+C_2$

$(0+50)^2(100)=(0+50)^3+C_2$

$(50)^2(100)=(50)^3+C_2$

$(50^2)(100)-50^3=C_2$

$50^2(100-50)=C_2$

$50^3=C_2$

$(t+50)^2A=(t+50)^3+50^3$

when $t=50$ min

$(50+50)^2A=(50+50)^3+50^3$

$A=\frac{100^3+50^3}{100^2}=112.5g$

Concentration: $\frac{112.5g}{200L}=0.5625\frac{g}{L}$

(4 points) 2. Initially 100 mg of a radioactive substance was present. After 6 hours, the mass had decreased by 3%. If the rate of decay is proportional to the amount of the substance present at time t, find the amount remaining after 24 hours. $A(0)=100$ $A(6)=97$

$\frac{dA}{dt}=kA$

$\frac{dA}{A}=k\,dt$

$\int\frac{dA}{A}=\int k\,dt$

$\ln|A|=kt+C$

$A=C_1e^{kt}$

$t=0$: $100=C_1e^{k(0)}$

$C_1=100$

$A=100e^{kt}$

$t=6$: $97=100e^{k(6)}$

$\frac{97}{100}=e^{6k}$

$\ln\frac{97}{100}=6k$

$\frac{1}{6}\ln\frac{97}{100}=k$

$A=100e^{(\frac{1}{6}\ln\frac{97}{100})k}$

when $t=24$

$A=100e^{(\frac{1}{6}\ln\frac{97}{100})24}$

$A\approx 88.5$ mg.

(4 points) *3.* Use Euler's method with the following function and given step size to find y_1, y_2, and y_3. Use at least four decimal places for your answers.

$$y' = -xy^2, \quad y(0) = 1, \quad h = 0.1$$

$x_0 = 0, \; y_0 = 1 \qquad f(x, y) = -xy^2$

$y_1 = 1 + 0.1(-0(1)^2) = 1$ $\quad (0.1, 1)$

$y_2 = 1 + 0.1(-(0.1)(1)^2) = 0.99$ $\quad (0.2, 0.99)$

$y_3 = 0.99 + 0.1(-(0.2)(0.99)^2) = 0.970398$ $\quad (0.3, 0.970398)$

(4 points) *4.* Consider a 100-volt electromotive force that is applied to an *RC*-series circuit in which the resistance is 5 ohms and the capacitance is 1/50 farad. Find the charge $q(t)$ on the capacitor if $q(0) = 0$.

$R\frac{dq}{dt} + \frac{1}{C}q = E(t)$

$5\frac{dq}{dt} + 50q = 100$

$\frac{dq}{dt} + 10q = 20$

$\mu(t) = e^{\int 10\,dt} = e^{10t}$

$\frac{d}{dt}\left[e^{10t}q\right] = 20e^{10t}$

$e^{10t}q = 2e^{10t} + C$

$q(0) = 0$

$e^0(0) = 2e^0 + C$

$0 = 2 + C$

$C = -2$

$e^{10t}q = 2e^{10t} - 2$

$q = 2 - 2e^{-10t}$