Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(6 points) 1. A tank contains 50 L of a solution into which 20 g of salt is dissolved. A solution containing 3 g/L of salt flows into the tank at a rate of 4 L/min, and the well-stirred mixture flows out at a rate of 2 L/min. What is the concentration of salt in the tank after 30 minutes?

$$V(0) = 50L, \quad A(0) = 20g \quad r_{in} = 4L/min, \quad c_{in} = 3g/L, \quad r_{out} = 2L/min$$

$$\frac{dV}{dt} = 4-2$$

$$dV = 2dt$$

$$V = 2t + C_1$$

$$50 = C_1$$

$$V = 2t + 50$$

$$\frac{dA}{dt} = \left(3\frac{g}{L}\right)\left(4\frac{L}{min}\right) - \left(\frac{A}{V}\frac{g}{L}\right)\left(2\frac{L}{min}\right)$$

$$\frac{dA}{dt} = 12 - \frac{2}{2t+50}A$$

$$\frac{dA}{dt} + \frac{1}{t+25}A = 12$$

$$\mu(t) = e^{\int \frac{1}{t+25}dt} = e^{\ln|t+25|} = t+25$$

$$(t+25)A = 12\int(t+25)\,dt$$

$$(t+25)A = 6(t+25)^2 + C_2$$

$$25(20) = 6(25)^2 + C_2$$

$$25(20) - 6(25)^2 = C_2$$

$$25(20 - 150) = C_2$$

$$25(-130) = C_2$$

$$(t+25)A = 6(t+25)^2 - 130(25)$$

$$t = 30$$

$$(55)A = 6(55)^2 - 130(25)$$

$$A = \frac{6(55)^2 - 130(25)}{55} \doteq 270.\overline{90}\,g$$

$$V = 2(30) + 50 = 110\,L$$

$$\text{Concentration} = \frac{270.\overline{90}\,g}{110\,L} = 2.46\,g/L$$

(5 points) **2.** Consider a 100-volt electromotive force that is applied to an *RC*-series circuit in which the resistance is 5 ohms and the capacitance is 1/50 farad. Find the charge $q(t)$ on the capacitor if $q(0) = 0$. Find the current, $i(t)$.

$$R\frac{dq}{dt} + \frac{1}{c}q = E(t)$$

$$5\frac{dq}{dt} + 50q = 100$$

$$\frac{dq}{dt} + 10q = 20$$

$$\mu(t) = e^{\int 10\,dt} = e^{10t}$$

$$e^{10t}q = 20\int e^{10t}\,dt$$

$$e^{10t}q = 2e^{10t} + C$$

$$q(0) = 0$$
$$0 = 2 + C$$
$$C = -2$$

$$e^{10t}q = 2e^{10t} - 2$$

$$q(t) = 2 - 2e^{-10t}$$

$$i(t) = 20e^{-10t}$$

(5 points) **3.** A small metal bar, whose initial temperature was 20°C, is dropped into a large container of water with a temperature of 80°C. How long will it take the bar to reach 50°C if it is known that its temperature in 5 seconds is 30°C?

$$T(t) = T_m + Ce^{kt}, \quad T(0) = 20, \quad T(5) = 30$$

$$T(t) = 80 + Ce^{kt}$$

$t=0, \ T=20$

$$20 = 80 + C$$

$$C = -60$$

$$T(t) = 80 - 60e^{kt}$$

$t=5, \ T=30$

$$30 = 80 - 60e^{5k}$$

$$-50 = -60e^{5k}$$

$$\frac{5}{6} = e^{5k}$$

$$\ln \tfrac{5}{6} = 5k$$

$$k = \tfrac{1}{5}\ln \tfrac{5}{6}$$

$$T(t) = 80 - 60e^{\left(\frac{1}{5}\ln \frac{5}{6}\right)t}$$

$$50 = 80 - 60e^{\left(\frac{1}{5}\ln \frac{5}{6}\right)t}$$

$$-30 = -60e^{\left(\frac{1}{5}\ln \frac{5}{6}\right)t}$$

$$\frac{1}{2} = e^{\left(\frac{1}{5}\ln \frac{5}{6}\right)t}$$

$$\ln \tfrac{1}{2} = \tfrac{1}{5}\ln\left(\tfrac{5}{6}\right)t$$

$$t = \frac{\ln \tfrac{1}{2}}{\tfrac{1}{5}\ln\left(\tfrac{5}{6}\right)}$$

$$t = 19.0 \text{ seconds.}$$