MATH 290 – QUIZ #3 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 10 points. Good luck!

(2 points) 1. Use Euler's method with the following function and the given step size to find $y_1, y_2,$ and y_3. Use at least four decimal places for your answers.

$$y' = 2xy^2, \quad y(0) = 1, \quad h = 0.1$$

$x_0 = 0 \quad y_0 = 1 \qquad y_1 = 1 + 0.1[2(0)(1)] = 1$

$x_1 = 0.1 \quad y_1 = 1 \qquad y_2 = 1 + 0.1[2(0.1)(1)^2] = 1.02$

$x_2 = 0.2 \quad y_2 = 1.02 \qquad y_3 = 1.02 + 0.1[2(0.2)(1.02)^2] = 1.061616$

(3 points) 2. A 20-volt electromotive force is applied to an *LR*-circuit in which the inductance is 0.1 henry and the resistance 4 ohms. If there is no current flowing initially, find the current, $i(t)$.

$$L\frac{di}{dt} + Ri = E$$

$$\frac{1}{10}\frac{di}{dt} + 4i = 20$$

$$\frac{di}{dt} + 40i = 200$$

$$\mu(t) = e^{\int 40\,dt} = e^{40t}$$

$$\frac{d}{dt}\left[e^{40t} i\right] = 200\, e^{40t}$$

$$e^{40t} i = 5e^{40t} + C$$

$$i = 5 + Ce^{-40t}$$

$$i(0) = 0 \qquad 0 = 5 + C$$

$$C = -5$$

$$i(t) = 5 - 5e^{-40t}$$

(5 points) 3. A tank initially contains 600 L of solution in which there is dissolved 1500 grams of chemical. A solution containing 5 g/L of the chemical flows into the tank at a rate of 6 L/min, and the well-stirred mixture flows out at a rate of 3 L/min. Determine the concentration of chemical in the tank after 30 minutes.

$V(0) = 600 \quad A(0) = 1500\,g$

$\frac{dV}{dt} = 6\frac{L}{min} - 3\frac{L}{min}$

$\frac{dV}{dt} = 3$

$dV = 3\,dt$

$V = 3t + C_1$

$600 = C_1$

$V = 3t + 600$

$\frac{dA}{dt} = C_{in} \cdot r_{in} - C_{out} \cdot r_{out}$

$\frac{dA}{dt} = 5\frac{g}{L} \cdot 6\frac{L}{min} - \frac{A}{V}\frac{g}{L} \cdot 3\frac{L}{min}$

$\frac{dA}{dt} = 30 - \frac{A}{3t+600} \cdot 3$

$\frac{dA}{dt} + \frac{1}{t+200} A = 30$

$\mu(t) = e^{\int \frac{1}{t+200}\,dt}$

$= e^{\ln|t+200|}$

$= t + 200$

$\frac{d}{dt}\left[(t+200)A\right] = 30(t+200)$

$(t+200)A = 15(t+200)^2 + C_2$

$(-200)(1500) = 15(-200)^2 + C_2$

$C_2 = -900000$

$(t+200)A = 15(t+200)^2 - 900000$

$t = 30\ min$

$(30+200)A = 15(30+200)^2 - 900000$

$A \doteq 2744.1\,g$

$V = 3(30) + 600 = 690\ L$

concentration:

$\frac{2744.1\,g}{690\,L} \doteq 3.98\frac{g}{L}$

(3 points) 4. An object whose initial temperature is 50°F is placed in a hot oven whose temperature is 450°F. After 20 minutes, the object's temperature is 150°F. Find the temperature of the object after 40 minutes.

$T = T_m + Ce^{kt}$

$T = 450 + Ce^{kt}$

$t = 0 \quad T = 50 \quad 50 = 450 + C$

$-400 = C$

$50 = 450 - 400e^{kt}$

$t = 20 \quad T = 150$

$150 = 450 - 400e^{k(20)}$

$-300 = -400e^{k(20)}$

$\frac{3}{4} = e^{k(20)}$

$\ln\frac{3}{4} = k(20)$

$\frac{1}{20}\ln\frac{3}{4} = k$

$t = 40$

$T = 450 - 400e^{(\frac{1}{20}\ln\frac{3}{4})(40)}$

$T = 225^\circ F$