MATH 290 – QUIZ #3 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

1. Given the following solution to the given differential equation.

$$y = c_1x^2 + c_2x^4 + 3; \qquad x^2y'' - 5xy' + 8y = 24$$

Determine if the following boundary conditions yield no solution, one solution, or infinitely many solutions.

(3 points) a. $y(-1) = 0,\ y(1) = 4$

$$\left.\begin{aligned} 0 &= c_1 + c_2 + 3 \\ 4 &= c_1 + c_2 + 3 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} c_1 + c_2 &= -3 \\ c_1 + c_2 &= 1 \end{aligned}\right\}$$

$\Rightarrow$ No solution

(3 points) b. $y(1) = 3,\ y(2) = 15$

$$\left.\begin{aligned} 3 &= c_1 + c_2 + 3 \\ 15 &= 4c_1 + 16c_2 + 3 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} c_1 + c_2 &= 0 \\ 4c_1 + 16c_2 &= 12 \end{aligned}\right\}$$

$$\begin{aligned} -4c_1 - 4c_2 &= 0 \\ 4c_1 + 16c_2 &= 12 \\ \hline 12c_2 &= 12 \\ c_2 &= 1 \Rightarrow c_1 = -1 \end{aligned}$$

$y = -x^2 + x^4 + 3$

One solution

(3 points) 2. Determine if the following set of functions are linearly independent or linearly dependent. Show work to justify your answer.

$$f_1(x) = e^{4x},\ f_2(x) = e^{2x}$$

$$\begin{vmatrix} e^{4x} & e^{2x} \\ 4e^{4x} & 2e^{2x} \end{vmatrix} = 2e^{6x} - 4e^{6x} \neq 0$$

$\therefore$ by the Wronskian, the functions are linearly independent on $(-\infty, \infty)$

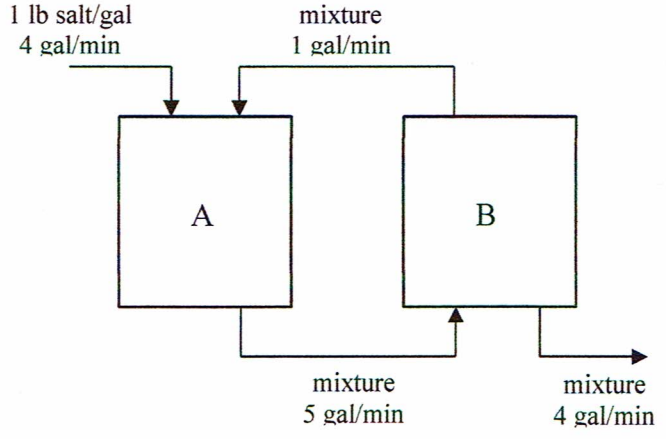
(5 points) 3. Given that $y_1(x) = e^{2x}$ is a solution to $y'' - 4y = 0$. Use reduction of order to find a second solution, $y_2(x)$.

$$y_2 = e^{2x}\int \frac{e^{-\int 0\,dx}}{(e^{2x})^2}\,dx = e^{2x}\int \frac{e^0}{e^{4x}}\,dx = e^{2x}\int e^{-4x}\,dx$$

$$= e^{2x}\,\frac{e^{-4x}}{-4} = -\frac{1}{4}e^{-2x}$$

$$\text{let } y_2 = e^{-2x}$$

(3 points) 4. Consider the two tanks shown in the given figure. Initially, tank A contains 100 gallons of water in which 50 pounds of salt is dissolved and tank B contains 100 gallons of pure water. Liquid is pumped in and out of the tanks as indicated in the figure. The mixture exchanged between the two tanks and the mixture pumped out of tank B are assumed to be well-stirred. Construct a mathematical model using a system of differential equations that describes rate of change of the number of pounds $x_1(t)$ and $x_2(t)$ of salt in tanks *A* and *B*, respectively, at time *t*.



$$\frac{dx_1}{dt} = \left(1\frac{lb}{gal}\right)\left(4\frac{gal}{min}\right) + \left(\frac{x_2}{100}\frac{lb}{gal}\right)\left(1\frac{gal}{min}\right) - \left(\frac{x_1}{100}\frac{lb}{gal}\right)\left(5\frac{gal}{min}\right) = 4 + \frac{x_2}{100} - \frac{x_1}{20}$$

$$\frac{dx_2}{dt} = \left(\frac{x_1}{100}\frac{lb}{gal}\right)\left(5\frac{gal}{min}\right) - \left(\frac{x_2}{100}\frac{lb}{gal}\right)\left(1\frac{gal}{min}\right) - \left(\frac{x_2}{100}\frac{lb}{gal}\right)\left(4\frac{gal}{min}\right) = \frac{x_1}{20} - \frac{x_2}{20}$$