MATH 290 – QUIZ #3 Name: KEY

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(5 points) *1.* Given that $y_1(x) = e^{3x}$ is a solution to $y'' + 2y' - 15y = 0$. Use reduction of order to find a second solution, $y_2(x)$.

$y_2 = uy_1 = ue^{3x}$

$y_2' = u'e^{3x} + 3ue^{3x}$

$y_2'' = u''e^{3x} + 6u'e^{3x} + 9ue^{3x}$

$y'' + 2y' - 15y = 0$

$(u''e^{3x} + 6u'e^{3x} + 9ue^{3x}) + 2(u'e^{3x} + 3ue^{3x}) - 15(ue^{3x}) = 0$

$u''e^{3x} + 8u'e^{3x} = 0$

let $w = u'$, $w' = u''$

$w'e^{3x} + 8we^{3x} = 0$

$w' + 8w = 0$

$\frac{dw}{dx} = -8w$

$\frac{dw}{w} = -8dx$

$\ln|w| = -8x + c_1$

$w = c_2e^{-8x}$

$u' = c_2e^{-8x}$

$u = -\frac{1}{8}c_2e^{-8x} + c_3$

$y_2 = e^{3x}(-\frac{1}{8}c_2e^{-8x} + c_3) = -\frac{1}{8}c_2e^{-5x} + c_3e^{3x}$

$y_2 = e^{-5x}$

OR

$y_2 = e^{3x}\int \frac{e^{-\int 2dx}}{(e^{3x})^2}dx = e^{3x}\int \frac{e^{-2x}}{e^{6x}}dx$

$= e^{3x}\int e^{-8x}dx = e^{3x}\frac{e^{-8x}}{-8} = -\frac{1}{8}e^{-5x}$

$y_2 = e^{-5x}$

(5 points) *2.* Verify that $y_1(x) = e^{4x}$ and $y_2(x) = xe^{4x}$ form a fundamental set of solutions to the differential equation $y'' - 8y' + 16y = 0$ on the interval $(-\infty, \infty)$.

1. Check if y_1 and y_2 are solutions:

$y = c_1e^{4x} + c_2xe^{4x}$

$y' = 4c_1e^{4x} + c_2e^{4x} + 4c_2xe^{4x}$

$y'' = 16c_1e^{4x} + 4c_2e^{4x} + 4c_2e^{4x} + 16c_2xe^{4x}$

$= 16c_1e^{4x} + 8c_2e^{4x} + 16c_2xe^{4x}$

$(16c_1e^{4x} + 8c_2e^{4x} + 16c_2xe^{4x}) - 8(4c_1e^{4x} + c_2e^{4x} + 4c_2xe^{4x}) + 16(c_1e^{4x} + c_2xe^{4x})$

$= 16c_1e^{4x} + 8c_2e^{4x} + 16c_2xe^{4x} - 32c_1e^{4x} - 8c_2e^{4x} - 32c_2xe^{4x} + 16c_1e^{4x} + 16c_2xe^{4x} = 0$ ✓

2. check that c_1 and c_2 are linearly independent

$\begin{vmatrix} e^{4x} & xe^{4x} \\ 4e^{4x} & e^{4x} + 4xe^{4x} \end{vmatrix} = e^{4x}(e^{4x} + 4xe^{4x}) - 4e^{4x}(xe^{4x})$

$= e^{8x} + 4xe^{8x} - 4xe^{8x} = e^{8x} \neq 0$ ∴ They are linearly independent.

∴ $y_1 = e^{4x}$ and $y_2 = xe^{4x}$ form a fundamental set of solutions.

3. Given the following solution to the given differential equation.

$$y = c_1 x^2 + c_2 x^4 + 3; \qquad x^2 y'' - 5xy' + 8y = 24$$

Determine if the following boundary conditions yield no solution, one solution, or infinitely many solutions.

(3 points) a. $y(-1) = 0,\ y(1) = 4$

$0 = 1c_1 + 1c_2 + 3$ $\qquad -3 = c_1 + c_2$ > contradiction

$4 = 1c_1 + 1c_2 + 3$ $\qquad 1 = c_1 + c_2$

$\therefore$ no solution

(3 points) b. $y(1) = 3,\ y(2) = 15$

$3 = 1c_1 + 1c_2 + 3$ $\qquad -4(c_1 + c_2 = 0)$

$15 = 4c_1 + 16c_2 + 3$ $\qquad 4c_1 + 16c_2 = 12$

$-4c_1 - 4c_2 = 0$

$4c_1 + 16c_2 = 12$

$12c_2 = 12$

$c_2 = 1$ $\qquad c_1 + 1 = 0$

$c_1 = -1$

$\therefore$ one solution.