

MATH 290 – QUIZ #4

Name: Key

**Directions:** Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(5 points) 1. Solve the following differential equation by using annihilators.

$$y'' - 2y' - 3y = 8e^{3x}$$

$$y_c = c_1 e^{3x} + c_2 e^{-x}$$

$$(D^2 - 2D - 3)y = 8e^{3x}$$

$$(D-3)(D+1)y = 0$$

$$y = c_1 e^{3x} + c_2 x e^{3x} + c_3 e^{-x}$$

$$y_p = A x e^{3x}$$

$$y_p = A x e^{3x}$$

$$y_p' = A e^{3x} + 3A x e^{3x}$$

$$y_p'' = 3A e^{3x} + 3A e^{3x} + 9A x e^{3x}$$

$$= 6A e^{3x} + 9A x e^{3x}$$

$$(6A e^{3x} + 9A x e^{3x}) - 2(A e^{3x} + 3A x e^{3x}) - 3(A x e^{3x}) = 8e^{3x}$$

$$6A e^{3x} + 9A x e^{3x} - 2A e^{3x} - 6A x e^{3x} - 3A x e^{3x} = 8e^{3x}$$

$$4A e^{3x} = 8e^{3x}$$

$$4A = 8$$

$$A = 2$$

$$y_p = 2x e^{3x}$$

$$y = c_1 e^{3x} + c_2 e^{-x} + 2x e^{3x}$$

2. Identify the annihilator for the following functions.

(2 points) a.  $7x^2 + 3e^{2x} - 4e^{-5x}$

$$D^3 (D-2)(D+5)$$

(2 points) b.  $3\cos 4x + 2\sin 3x$

$$(D^2 + 16)(D^2 + 9)$$

(2 points) c.  $x e^{4x} \sin 2x$

$$(D^2 - 2.4D + (16+4))^2$$

$$= (D^2 - 8D + 20)^2$$

(5 points) 3. Solve the following differential equation by using the superposition approach.

$$y'' - y' - 2y = 10 \sin 2x$$

$$y_c = c_1 e^{2x} + c_2 e^{-x}$$

$$y_p = A \cos 2x + B \sin 2x$$

$$y_p' = -2A \sin 2x + 2B \cos 2x$$

$$y_p'' = -4A \cos 2x - 4B \sin 2x$$

$$\begin{aligned} (-4A \cos 2x - 4B \sin 2x) - (-2A \sin 2x + 2B \cos 2x) - 2(A \cos 2x + B \sin 2x) &= 10 \sin 2x \\ -4A \cos 2x - 4B \sin 2x + 2A \sin 2x - 2B \cos 2x - 2A \cos 2x - 2B \sin 2x &= 10 \sin 2x \end{aligned}$$

$$(-6A - 2B) \cos 2x + (2A - 6B) \sin 2x = 10 \sin 2x$$

$$3(2A - 6B = 10)$$

$$-6A - 2B = 0$$

$$6A - 18B = 30$$

$$\begin{array}{r} -6A - 2B = 0 \\ \hline -20B = 30 \end{array}$$

$$-20B = 30$$

$$B = -\frac{3}{2}$$

$$-6A - 2\left(-\frac{3}{2}\right) = 0$$

$$-6A + 3 = 0$$

$$A = \frac{1}{2}$$

$$y_p = \frac{1}{2} \cos 2x - \frac{3}{2} \sin 2x$$

$$y = c_1 e^{2x} + c_2 e^{-x} + \frac{1}{2} \cos 2x - \frac{3}{2} \sin 2x$$