***MATH 290 – QUIZ #4*** *Name:* KEY

**Directions:** Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

*(5 points) 1.* Solve the following differential equation by using the superposition approach.

$$y'' + y' - 2y = x^2 - 2x$$

$y'' + y' - 2y = 0$

$m^2 + m - 2 = 0$

$(m+2)(m-1) = 0$

$m = -2, 1$

$y_c = c_1 e^{-2x} + c_2 e^{x}$

$y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$(2A) + (2Ax + B) - 2(Ax^2 + Bx + C) = x^2 - 2x$

$-2Ax^2 + (2A - 2B)x + (2A + B - 2C) = x^2 - 2x$

$-2A = 1 \quad 2A - 2B = -2 \quad 2A + B - 2C = 0$

$A = -1/2 \quad -1 - 2B = -2 \quad -1 + 1/2 - 2C = 0$

$-2B = -1 \quad -2C = 1/2$

$B = 1/2 \quad C = -1/4$

$y_p = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{4}$

$y = c_1 e^{-2x} + c_2 e^{x} - \frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{4}$

*(5 points) 2.* Solve the following differential equation by using annihilators.

$$y'' - 5y' + 4y = 6e^{4x}$$

$y'' - 5y' + 4y = 0$

$m^2 - 5m + 4 = 0$

$(m-4)(m-1) = 0$

$m = 4, 1$

$y_c = c_1 e^{4x} + c_2 e^{x}$

$y_p = Axe^{4x}$

$y_p' = Ae^{4x} + 4xe^{4x}$

$y_p'' = 4Ae^{4x} + 4e^{4x} + 16xe^{4x}$

$8Ae^{4x} + 16xe^{4x} - 5(Ae^{4x} + 4xe^{4x}) + 4(Axe^{4x}) = 6e^{4x}$

$8Ae^{4x} + 16xe^{4x} - 5Ae^{4x} - 20xe^{4x} + 4Axe^{4x} = 6e^{4x}$

$3Ae^{4x} = 6e^{4x}$

$A = 2$

$y_p = 2xe^{4x}$

Annihilator of $e^{4x}$ is $D - 4$

$(D^2 - 5D + 4)y = 6e^{4x}$

$(D-4)(D-1)y = 6e^{4x}$

$(D-4)^2(D-1)y = 0$

$y = c_1 e^{4x} + c_2 xe^{4x} + c_3 e^{x}$

$y = c_1 e^{4x} + c_2 e^{x} + 2xe^{4x}$

*(6 points)* 3. Solve the following differential equation by using variation of parameters.

$$y'' + y = \csc x$$

$y'' + y = 0$

$m^2 + 1 = 0$

$m = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$

$y_p = u_1 \cos x + u_2 \sin x$

$( \quad u_1' \cos x + u_2' \sin x = 0 \quad ) \sin x$

$(-u_1' \sin x + u_2' \cos x = \csc x \quad ) \cos x$

$u_1' \cos x \sin x + u_2' \sin^2 x = 0$

$-u_1' \sin x \cos x + u_2' \cos^2 x = \cot x$

$u_2' = \cot x$

$u_2 = \int \cot x \, dx$

$u_2 = \ln|\sin x|$

$u_1' \cos x + \cot x \sin x = 0$

$u_1' \cos x = -\cos x$

$u_1' = -1$

$u_1' = -\int dx$

$u_1 = -x$

$y_p = -x \cos x + \ln|\sin x| \sin x$

$y = c_1 \cos x + c_2 \sin x - x \cos x + \sin x \ln|\sin x|$