MATH 290 – QUIZ #4 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(5 points) 1. Solve the following differential equation by using annihilators.

$$y'' + 2y' - 8y = 4e^{2x}$$

$y'' + 2y' - 8y = 0$

$(D^2 + 2D - 8)y = 0$

$(D+4)(D-2)y = 0$

$y_c = c_1 e^{-4x} + c_2 e^{2x}$

$(D+4)(D-2)y = 4e^{2x}$

$(D+4)(D-2)^2 y = 0$

$y = c_1 e^{-4x} + c_2 e^{2x} + c_3 x e^{2x}$

$y_p = Axe^{2x}$

$y_p' = Ae^{2x} + 2Axe^{2x}$

$y_p'' = 2Ae^{2x} + 2Ae^{2x} + 4Axe^{2x}$

$= 4Ae^{2x} + 4Axe^{2x}$

$(4Ae^{2x} + 4Axe^{2x}) + 2(Ae^{2x} + 2Axe^{2x}) - 8(Axe^{2x}) = 4e^{2x}$

$6Ae^{2x} = 4e^{2x}$

$6A = 4$

$A = 2/3$

$y_p = 2/3\, xe^{2x}$

$y = c_1 e^{-4x} + c_2 e^{2x} + \frac{2}{3} x e^{2x}$

(5 points) 2. Solve the following differential equation by using variation of parameters.

$$y'' + y = \csc x$$

$y'' + y = 0$

$(D^2 + 1)y = 0$

$y_c = c_1 \cos x + c_2 \sin x$

$y_p = u_1 \cos x + u_2 \sin x$

$u_1' \cos x + u_2' \sin x = 0$

$u_1'(-\sin x) + u_2' \cos x = \csc x$

$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$= \cos^2 x + \sin^2 x = 1$

$u_1' = \begin{vmatrix} 0 & \sin x \\ \csc x & \cos x \end{vmatrix} = -1$

$u_1 = -\int dx = -x$

$u_2' = \begin{vmatrix} \cos x & 0 \\ -\sin x & \csc x \end{vmatrix} = \cot x$

$u_2 = \int \cot x \, dx$

$= \ln|\sin x|$

$y_p = -x\cos x + \sin x \ln|\sin x|$

$y = c_1 \cos x + c_2 \sin x - x\cos x + \sin x \ln|\sin x|$

(6 points) *3.* Solve the following differential equation by using variation of parameters.

$$x^2y'' - 2xy' + 2y = x^4e^x \quad \rightarrow \quad y'' - \frac{2y'}{x} + \frac{2}{x^2}y = x^2e^x \leftarrow f(x)$$

$m^2 + (-2-1)m + 2 = 0$

$m^2 - 3m + 2 = 0$

$(m-2)(m-1) = 0$

$m = 2, 1$

$y_c = c_1x^2 + c_2x$

$y_p = u_1x^2 + u_2x$

$u_1'x^2 + u_2'x = 0$

$u_1'(2x) + u_2'(1) = x^2e^x$

$$W = \begin{vmatrix} x^2 & x \\ 2x & 1 \end{vmatrix} = x^2 - 2x^2 = -x^2$$

$$u_1' = \frac{\begin{vmatrix} 0 & x \\ x^2e^x & 1 \end{vmatrix}}{-x^2} = \frac{-x^3e^x}{-x^2} = xe^x$$

$u_1 = \int xe^x dx = xe^x - \int e^x dx = xe^x - e^x$

$u = x \quad dv = e^x$

$du = dx \quad v = e^x$

$$u_2' = \frac{\begin{vmatrix} x^2 & 0 \\ 2x & x^2e^x \end{vmatrix}}{-x^2} = \frac{x^4e^x}{-x^2} = -x^2e^x$$

$u_2 = -\int x^2e^x dx = -\left[x^2e^x - \int 2xe^x dx\right] = -\left[x^2e^x - \left(2xe^x - \int 2e^x dx\right)\right]$

$u = x^2 \quad dv = e^x$

$du = 2x \quad v = e^x$

$u = 2x \quad dv = e^x$

$du = 2 \quad v = e^x$

$= -\left[x^2e^x - 2xe^x + 2e^x\right] = -x^2e^x + 2xe^x - 2e^x$

$y_p = (xe^x - e^x)x^2 + (-x^2e^x + 2xe^x - 2e^x)x$

$= x^3e^x - x^2e^x - x^3e^x + 2x^2e^x - 2xe^x = x^2e^x + 2xe^x$

$y = c_1x^2 + c_2x + x^2e^x - 2xe^x$