MATH 290 – QUIZ #5 *Name:* KEY

Directions: Please show all work for maximum credit. There are 16 points on this quiz. Good luck!

(2 points) 1. Solve the following differential equation.

$$3x^2y'' - 2xy' + 2y = 0$$

$3m^2 + (2-3)m + 2 = 0$

$3m^2 - 5m + 2 = 0$

$(3m - 2)(m - 1) = 0$

$m = 2/3, 1$

$y = C_1 x^{2/3} + C_2 x$

(5 points) 3. Solve the following differential equation by using variation of parameters.

$$y'' + y = \sec x$$

$m^2 + 1 = 0, \quad m = \pm i$

$y_c = C_1 \cos x + C_2 \sin x$

$y_p = u_1 \cos x + u_2 \sin x$

$u_1' \cos x + u_2' \sin x = 0$

$-u_1' \sin x + u_2' \cos x = \sec x$

$$\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$u_1' = \frac{\begin{vmatrix} 0 & \sin x \\ \sec x & \cos x \end{vmatrix}}{1} = -\tan x$$

$u_1 = -\int \tan x \, dx = \ln|\cos x|$

$$u_2' = \frac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \end{vmatrix}}{1} = 1$$

$u_2 = \int dx = x$

$y_p = \cos x \ln|\cos x| + x \sin x$

$y = C_1 \cos x + C_2 \sin x + \cos x \ln|\cos x| + x \sin x$

(4 points) 3. A mass weighing 20 pounds, attached to the end of a spring, stretches it 6 inches. Initially, the mass is released from rest from a point 6 inches below the equilibrium position. Find the equation of motion, $x(t)$.

$F = mg$

$20 = m(32)$

$\frac{5}{8}\text{ slug} = m$

$F = kx$

$20 = k\left(\frac{1}{2}\right)$

$40\frac{\text{lb}}{\text{ft}} = k$

$\frac{5}{8}\frac{d^2x}{dt^2} + 40x = 0$

$\frac{d^2x}{dt^2} + 64x = 0$

$m_1^2 + 64 = 0$

$m_1 = \pm 8i$

$x(t) = c_1\cos 8t + c_2\sin 8t$

$x'(t) = -8c_1\sin 8t + 8c_2\cos 8t$

$x(0) = \frac{1}{2}\text{ ft}, \quad x'(0) = 0\text{ ft/s}$

$x(0)$: $c_1 = \frac{1}{2}$

$x'(0)$: $8c_2 = 0$

$c_2 = 0$

$x(t) = \frac{1}{2}\cos 8t$

(5 points) 4. A mass weighing 3.2 pounds stretches a spring 1.6 feet. The entire system is placed in a medium that offers a damping force that is numerically equal to 0.4 times the instantaneous velocity. Find the equation of motion, $x(t)$, if the mass is initially released from rest from a point a point 1 foot above the equilibrium position. What type of motion is this?

$F = mg$

$3.2 = m(32)$

$0.1\text{ slug} = m$

$F = kx$

$3.2 = k(1.6)$

$2\frac{\text{lb}}{\text{ft}} = k$

$0.1\frac{d^2x}{dt^2} + 0.4\frac{dx}{dt} + 2x = 0$

$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 20x = 0$

$m_1^2 + 4m_1 + 20 = 0$

$m_1 = \frac{-4 \pm \sqrt{16 - 80}}{2}$

$= \frac{-4 \pm \sqrt{-64}}{2}$

$= \frac{-4 \pm 8i}{2}$

$= -2 \pm 4i$

$x(t) = e^{-2t}[c_1\cos 4t + c_2\sin 4t]$

$x'(t) = -2e^{-2t}[c_1\cos 4t + c_2\sin 4t] + e^{-2t}[-4c_1\sin 4t + 4c_2\cos 4t]$

$x(0) = -1\text{ ft} \quad x'(0) = 0\text{ ft/s}$

$x(0)$: $c_1 = -1$

$x'(0)$: $2 + 4c_2 = 0$

$c_2 = -\frac{1}{2}$

$x(t) = e^{-2t}\left[-\cos 4t - \frac{1}{2}\sin 4t\right]$