*MATH 290 – QUIZ #5* *Name:* KEY

**Directions:** This is a take home quiz. This quiz is due at the beginning of class on Monday, February 11, 2019. Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

*(3 points) 1.* Solve the following differential equation.

$$x^2y'' + 3xy' - 4y = 0$$

$m^2 + (3-1)m - 4 = 0$

$m^2 + 2m - 4 = 0$

$m = \dfrac{-2 \pm \sqrt{4 - 4(1)(-4)}}{2}$

$= \dfrac{-2 \pm \sqrt{20}}{2}$

$= \dfrac{-2 \pm 2\sqrt{5}}{2}$

$= -1 \pm \sqrt{5}$

$y = C_1 x^{-1+\sqrt{5}} + C_2 x^{-1-\sqrt{5}}$

*(4 points) 2.* Solve the following differential equation by variation of parameters.

$$x^2y'' - 2xy' + 2y = x^4e^x$$

$m^2 + (-2-1)m + 2 = 0$

$m^2 - 3m + 2 = 0$

$m = 2, 1$

$y_c = C_1 x^2 + C_2 x$

$y_p = u_1 x^2 + u_2 x$

$u_1' x^2 + u_2' x = 0$

$-x\,( u_1'(2x) + u_2'(1) = x^2 e^x)$

$u_1' x^2 + u_2' x = 0$

$-u_1' 2x^2 - u_2' x = -x^3 e^x$

$-x^2 u_1' = -x^3 e^x$

$u_1' = x e^x$

$u_1' = x e^x$

$u_1 = x e^x - e^x$

$u_1' x^2 + u_2' x = 0$

$x^3 e^x + u_2' x = 0$

$u_2' x = -x^3 e^x$

$u_2' = -x^2 e^x$

| | |
|---|---|
| $-x^2$ | $e^x$ |
| $-2x$ | $e^x$ |
| $-2$ | $e^x$ |
| $0$ | $e^x$ |

$u_2 = -x^2 e^x + 2x e^x - 2e^x$

$y_p = (xe^x - e^x)x^2 + (-x^2e^x + 2xe^x - 2e^x)x$

$= x^3e^x - x^2e^x - x^3e^x + 2x^2e^x - 2xe^x$

$= x^2e^x - 2xe^x$

$y = C_1 x^2 + C_2 x + x^2 e^x - 2xe^x$

*(5 points)* *3.* A mass weighing 24 pounds, attached to the end of a spring, stretches it 4 inches. Initially, the mass is released from rest with a downward velocity of 2 ft/s from a point 3 inches above the equilibrium position. Find the equation of motion, $x(t)$. $x(0) = -\frac{1}{4}$ $x'(0) = 2$

$F = kx$

$24 = k \cdot \frac{1}{3}$

$72 = k$

$24 = m(32)$

$\frac{24}{32} = m$

$\frac{3}{4} = m$

$\frac{3}{4}\frac{d^2x}{dt^2} + 72x = 0$

$\frac{d^2x}{dt^2} + 96x = 0$

$n^2 + 96 = 0$

$n = \pm 4i\sqrt{6}$

$t = 0$ $x(0) = -\frac{1}{4}$

$c_1 = -\frac{1}{4}$

$x'(0) = 2$

$2 = 4\sqrt{6}\, c_2$

$c_2 = \frac{1}{2\sqrt{6}}$

$x = c_1 \cos 4\sqrt{6}t + c_2 \sin 4\sqrt{6}t$

$x' = -4\sqrt{6}c_1 \sin 4\sqrt{6}t + 4\sqrt{6}c_2 \cos 4\sqrt{6}t$

$x(t) = -\frac{1}{4}\cos 4\sqrt{6}t + \frac{1}{2\sqrt{6}} \sin 4\sqrt{6}t$

*(5 points)* *4.* After a mass weighing 10 pounds is attached to a 5-foot spring, the spring measures 7 feet. The mass is removed and replaced with another mass that weighs 8 pounds. The entire system is placed in a medium that offers a damping force that is numerically equal to the instantaneous velocity. Find the equation of motion, $x(t)$, if the mass is initially released from a point ½ foot below the equilibrium position with a downward velocity of 1 ft/s. $x(0) = \frac{1}{2}$

$x'(0) = 1$

$10 = k \cdot 2$

$k = 5$

$8 = m \cdot 32$

$\frac{1}{4} = m$

$\frac{1}{4}\frac{d^2x}{dt^2} + \frac{dx}{dt} + 5x = 0$

$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 20x = 0$

$n^2 + 4n + 20 = 0$

$n = \frac{-4 \pm \sqrt{16 - 4(1)(20)}}{2(1)}$

$n = \frac{-4 \pm \sqrt{16-80}}{2}$

$= \frac{-4 \pm \sqrt{-64}}{2}$

$= \frac{-4 \pm 8i}{2}$

$= -2 \pm 4i$

$x = e^{-2t}\left(\frac{1}{2}\cos 4t + \frac{1}{2}\sin 4t\right)$

$x = e^{-2t}[c_1 \cos 4t + c_2 \sin 4t]$

$x' = e^{-2t}[-4c_1 \sin 4t + 4c_2 \cos 4t] + -2e^{-2t}[c_1 \cos 4t + c_2 \sin 4t]$

$x(0) = 1/2$ $1/2 = c_1$

$x'(0) = 1$ $1 = 4c_2 - 2c_1 \Rightarrow 1 = 4c_2 - 1 \Rightarrow 2 = 4c_2$

$c_2 = 1/2$