*MATH 290 – QUIZ #5* Name: KEY

**Directions:** Please show all work for maximum credit. There are 16 points on this quiz. Good luck!

*(5 points) 1.* A mass weighing 12 pounds, attached to the end of a spring, stretches it 2 feet. Initially, the mass is released from a point 1 foot below the equilibrium position with an upward velocity of 4 ft/s. Find the equation of motion, $x(t)$, and find the equation of motion as a single sine function.

$F = kx$

$12 = k \cdot 2$

$6 \frac{lb}{ft} = k$

$x(0) = 1 ft$

$x'(0) = -4 ft/s$

$W = mg$

$12 = m(32)$

$\frac{3}{8}$ slug $= m$

$\frac{3}{8}\frac{d^2x}{dt^2} + 6x = 0$

$\frac{d^2x}{dt^2} + 16x = 0$

$x(t) = c_1 \cos 4t + c_2 \sin 4t$

$x(0) = 1 = c_1$

$x(t) = \cos 4t + c_2 \sin 4t$

$x'(0) = -4\sin 4t + 4c_2 \sin 4t$

$x'(0) = -4$

$-4 = 4c_2$

$c_2 = -1$

$x(t) = \cos 4t - \sin 4t$


$\tan \varphi = -1$

$\varphi = 3\pi/4$

$A = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$x(t) = \sqrt{2} \sin\left(4t + \frac{3\pi}{4}\right)$

*(5 points) 2.* Find the charge on the capacitor in an *LRC*-series circuit when the inductance is 1/4 henry, the resistance is 20 ohms, the capacitance is 1/300 farad, and the electromotive force is $E(t) = 0$ V. The initial charge on the capacitor is $q(0) = 4$ C and the initial current is $i(0) = 0$ A.

$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$

$\frac{1}{4}\frac{d^2q}{dt^2} + 20\frac{dq}{dt} + 300q = 0$

$\frac{d^2q}{dt^2} + 80\frac{dq}{dt} + 1200q = 0$

$m^2 + 80m + 1200 = 0$

$m = \frac{-80 \pm \sqrt{80^2 - 4(1200)}}{2} = \frac{-80 \pm \sqrt{6400 - 4800}}{2}$

$= \frac{-80 \pm \sqrt{1600}}{2} = \frac{-80 \pm 40}{2}$

$m = -60, -20$

$q(t) = c_1 e^{-60t} + c_2 e^{-20t}$

$q'(t) = -60c_1 e^{-60t} - 20c_2 e^{-20t}$

$q(0) = 0, \quad q'(0) = i(0) = 0$

$20\,(c_1 + c_2 = 4) \Rightarrow 20c_1 + 20c_2 = 80$

$-60c_1 - 20c_2 = 0 \qquad -60c_1 - 20c_2 = 0$

$-40c_1 = 80$

$c_1 = -2$

$-2 + c_2 = 4$

$c_2 = 6$

$q(t) = -2e^{-60t} + 6e^{-20t}$

*(6 points)* *3.* A force of 0.2 lb stretches a spring 6 inches. With one end fixed, a mass weighing 6.4 pounds is attached to the other end of the spring. The entire system is placed in a medium that offers a damping force that is numerically equal to 1.2 times the instantaneous velocity. Find the equation of motion, $x(t)$, if the mass is initially released from rest from a point 6 inches above the equilibrium position. What type of motion is this?

$F = kx$

$0.2\,lb = k(0.5\,ft)$

$0.4\,\frac{lb}{ft} = k$

$W = mg$

$6.4\,lb = m(32\,ft/s^2)$

$0.2\,slug = m$

$x(0) = -\frac{1}{2}\,ft$

$x'(0) = 0\,ft/s$

$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = 0$

$0.2\frac{d^2x}{dt^2} + 1.2\frac{dx}{dt} + 0.4x = 0$

$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 2x = 0$

$r^2 + 6r + 2 = 0$

$r = \frac{-6 \pm \sqrt{36 - 4(2)}}{2}$

$= \frac{-6 \pm \sqrt{28}}{2}$

$= \frac{-6 \pm 2\sqrt{7}}{2} = -3 \pm \sqrt{7}$

$x(t) = \left(\frac{-3-\sqrt{7}}{4\sqrt{7}}\right)e^{(-3+\sqrt{7})t} + \left(\frac{3-\sqrt{7}}{4\sqrt{7}}\right)e^{(-3-\sqrt{7})t}$

$x(t) \doteq (-0.533)e^{(-3+\sqrt{7})t} + (0.0335)e^{(-3-\sqrt{7})t}$

overdamped

$x(t) = c_1 e^{(-3+\sqrt{7})t} + c_2 e^{(-3-\sqrt{7})t}$

$x'(t) = (-3+\sqrt{7})c_1 e^{(-3+\sqrt{7})t} + (-3-\sqrt{7})c_2 e^{(-3-\sqrt{7})t}$

$x(0) = -\frac{1}{2}$

$-\frac{1}{2} = c_1 + c_2$

$c_1 = -c_2 - \frac{1}{2}$

$x'(0) = 0$

$0 = (-3+\sqrt{7})c_1 + (-3-\sqrt{7})c_2$

$0 = (-3+\sqrt{7})(-c_2 - \frac{1}{2}) + (-3-\sqrt{7})c_2$

$0 = -c_2(-3+\sqrt{7}) - \left(\frac{-3+\sqrt{7}}{2}\right) + (-3-\sqrt{7})c_2$

$c_2(-3+\sqrt{7}) + (3+\sqrt{7})c_2 = \frac{3-\sqrt{7}}{2}$

$c_2 = \frac{3-\sqrt{7}}{4\sqrt{7}}$

$c_1 = -\left(\frac{3-\sqrt{7}}{4\sqrt{7}}\right) - \frac{1}{2} = \frac{-3+\sqrt{7}}{4\sqrt{7}} - \frac{1}{2}$

$= \frac{-3+\sqrt{7}-2\sqrt{7}}{4\sqrt{7}} = \frac{-3-\sqrt{7}}{4\sqrt{7}}$