Question #1

a) $5x+1 \geq 3x-9$

$5x+1-3x \geq -9$

$2x+1 \geq -9$

$2x \geq -9-1$

$\frac{2x}{2} \geq \frac{-10}{2}$

$x \geq -5$ $[-5, \infty)$

b) $4x-(6x+1) \leq 8x+2(x-3)$

$4x-6x-1 \leq 8x+2x-6$

$-2x-1 \leq 10x-6$

$-2x+10x \leq -6$

$-12x-1 \leq -6$

$-12x \leq -6+1$

$-12x \leq -5$

$\frac{-12x}{-12} \leq \frac{-5}{-12}$

$x \geq 5/12$ $[5/12, \infty)$

c) $-w+12+9w > 2w+9+w$

$8w+12 > 2w+9$

$8w+12-2w > 9$

$6w+12 > 9$

$6w > 9-12$

$\frac{6w}{6} > \frac{-3}{6}$

$w > -1/2$ $(-1/2, \infty)$

d) $2(x-5)+3x < 4(x+6)+1$

$2x-10+3x < 4x+24+1$

$5x-10 < 4x+23$

$5x-10-4x < 23$

$x-10 < 23$

$x < 23+10$

$x < 33$ $(-\infty, 33)$

Question #2

a) $(-2b^6+3b^4-b^2)+(b^6+2b^4+2b^2) = \boxed{-b^6+5b^4+b^2}$

b) $(5x^2y-2xy+9xy^2)-(8x^2y+13xy+12xy^2)$

$(5x^2y-2xy+9xy^2)+(-8x^2y-13xy-12xy^2)$

$\boxed{-3x^2y-15xy-3xy^2}$

c) $[(6t^2-3t+1)-(12t^2+2t-6)]-[(4t^2-3t-8)+(-6t^2+10t+12)]$

$[(6t^2-3t+1)+(-12t^2-2t+6)]-[(4t^2-3t-8)+(-6t^2+10t-12)]$

$(-6t^2-5t+7)-(-2t^2+7t-20)$

$(-6t^2-5t+7)+(2t^2-7t+20) = \boxed{-4t^2-12t+27}$

Question #1 e- at end

Question #3

a) $(5x+3y)(6x-5y)$

$30x^2-25xy+18xy-15y^2$

$\boxed{30x^2-7xy-15y^2}$

b) $(9y-2)(8y^2-6y+1)$

$72y^3-54y^2+9y-16y^2+12y-2$

$\boxed{72y^3-70y^2+21y-2}$

c) $5t^4(2t-3)(6t+5)$

$5t^4(12t^2+10t-18t-15)$

$5t^4(12t^2-8t-15)$

$\boxed{60t^6-40t^5-75t^4}$

d) $(4x-3)^2 = (4x-3)(4x-3)$

$16x^2-12x-12x+9$

$\boxed{16x^2-24x+9}$

e) $(6x+5)^2 = (6x+5)(6x+5)$

$36x^2+30x+30x+25$

$\boxed{36x^2+60x+25}$

f) $(2x-5)^3 = (2x-5)(2x-5)(2x-5)$

$(4x^2-10x-10x+25)(2x-5)$

$(4x^2-20x+25)(2x-5)$

$8x^3-20x^2-40x^2+100x+50x-125$

$\boxed{8x^3-60x^2+150x-125}$

g) $(3x-5y)(2x+7y)$

$6x^2+21xy-10xy-35y^2$

$\boxed{6x^2+11xy-35y^2}$

Question #4

a.) $\frac{5t^8+5t^7+15}{5t}$

$\frac{5t^8}{5t}+\frac{5t^7}{5t}+\frac{15}{5t}$

$\boxed{t^7+t^6+\frac{3}{t}}$

b.) $\frac{16x^6+24x^4+10x^3}{4x^5}$

$\frac{16x^6}{4x^5}+\frac{24x^4}{4x^5}+\frac{10x^3}{4x^5}$

$\boxed{4x+\frac{6}{x}+\frac{5}{2x^2}}$

c. $(x^3-2x^2+2x-5)\div(x+1)$

$$\begin{array}{r} x^2-3x+5 \\ x+1\overline{)\,x^3-2x^2+2x-5} \\ -(x^3+x^2) \\ \hline -3x^2+2x \\ -(-3x^2-3x) \\ \hline 5x-5 \\ -(5x+5) \\ \hline -10 \end{array}$$

$\frac{x^3}{x}=x^2$

$x^2(x+1)=x^3+x^2$

$(x^3-2x^2)-(x^3+x^2)$

$(x^3-2x^2)+(-x^3-x^2)$

$-3x^2$

$\frac{-3x^2}{x}=-3x$

$-3x(x+1)=-3x^2-3x$

$(-3x^2+2x)-(-3x^2-3x)$

$(-3x^2+2x)+(3x^2+3x)$

$5x$

$\frac{5x}{x}=5$

$5(x+1)=5x+5$

$(5x-5)-(5x+5)$

$(5x-5)+(-5x-5)$

-10

$\boxed{x^2-3x+5+\frac{-10}{x+1}}$

c. $(t^2+2t-35)\div(t-5)$

$$\begin{array}{r} t+7 \\ t-5\overline{)\,t^2+2t-35} \\ -(t^2-5t) \\ \hline 7t-35 \\ -(7t-35) \\ \hline 0 \end{array}$$

$\boxed{t+7}$

$\frac{t^2}{t}=t$

$t(t-5)=t^2-5t$

$(t^2+2t)-(t^2-5t)$

$(t^2+2t)+(-t^2+5t)=7t$

$\frac{7t}{t}=7$

$7(t-5)=7t-35$

d.) $\frac{6r^4-11r^3-r^2+16r-8}{2r-3}$

$\boxed{3r^3-r^2-2r+5+\frac{7}{2r-3}}$

$$\begin{array}{r} 3r^3-r^2-2r+5 \\ 2r-3\overline{)\,6r^4-11r^3-r^2+16r-8} \\ -(6r^4-9r^3) \\ \hline -2r^3-r^2 \\ -(-2r^3+3r^2) \\ \hline -4r^2+16r \\ -(-4r^2+6r) \\ \hline 10r-8 \\ -(10r-15) \\ \hline 7 \end{array}$$

$\frac{6r^4}{2r}=3r^3$

$3r^3(2r-3)$

$6r^4-9r^3$

$(6r^4-11r^3)-(6r^4-9r^3)$

$(6r^4-11r^3)+(-6r^4+9r^3)=-2r^3$

$\frac{-2r^3}{2r}=-r^2$

$-r^2(2r-3)=-2r^3+3r^2$

$(-2r^3-r^2)-(-2r^3+3r^2)$

$(-2r^3-r^2)+(2r^3-3r^2)=-4r^2$

$\frac{-4r^2}{2r}=-2r$

$-2r(2r-3)=-4r^2+6r$

$(-4r^2+16r)-(-4r^2+6r)$

$(-4r^2+16r)+(4r^2-6r)=10r$

$\frac{10r}{2r}=5$

$5(2r-3)=10r-15$

$(10r-8)-(10r-15)$

$(10r-8)+(-10r+15)=7$

Question #4 continued

e.) $(3x^4+2x^3-11x^2-2x+5) \div (3x-4)$

$$3x-4 \;\overline{)\; 3x^4+2x^3-11x^2-2x+5}$$

x^3+2x^2-x-2

$-(3x^4-4x^3)$

$6x^3-11x^2$

$-(6x^3-8x^2)$

$-3x^2-2x$

$-(-3x^2+4x)$

$-6x+5$

$-(-6x+8)$

-3

$$\boxed{x^3+2x^2-x-2+\frac{-3}{3x-4}}$$

$\frac{3x^4}{3x}=x^3$ $\quad x^3(3x-4)=3x^4-4x^3$

$(3x^4+2x^3)-(3x^4-4x^3)$

$(3x^4+2x^3)+(-3x^4+4x^3)=6x^3$

$\frac{6x^3}{3x}=2x^2$ $\quad 2x^2(3x-4)=6x^3-8x^2$

$(6x^3-11x^2)-(6x^3-8x^2)$

$(6x^3-11x^2)+(-6x^3+8x^2)=-3x^2$

$\frac{-3x^2}{3x}=-x$ $\quad -x(3x-4)=-3x^2+4x$

$(-3x^2-2x)-(-3x^2+4x)$

$(-3x^2-2x)+(3x^2-4x)=-6x$

$\frac{-6x}{3x}=-2$ $\quad -2(3x-4)=-6x+8$

$(-6x+5)-(-6x+8)=(-6x+5)+(6x-8)=-3$

Question #5

a.) $s^2-6s-27$ $\quad -9\cdot 3=-27$, $-9+3=-6$

$\boxed{(s-9)(s+3)}$

b.) $m^2-12m+36$ $\quad -6\cdot -6=36$, $-6+-6=-12$

$(m-6)(m-6)$

$\boxed{(m-6)^2}$

c.) $15x^2-14x-8$ $\quad -20\cdot 6=-120$, $-20+6=-14$

$15x^2-20x+6x-8$

$(15x^2-20x)+(6x-8)$

$5x(3x-4)+2(3x-4)$

$\boxed{(3x-4)(5x+2)}$

d.) $12x^3-2x^2y-24xy^2$

$2x(6x^2-xy-12y^2)$ $\quad -9\cdot 8=-72$, $-9+8=-1$

$\boxed{2x(2x-3y)(3x+4y)}$

$6x^2-9xy+8xy-12y^2$

$(6x^2-9xy)+(8xy-12y^2)$

$3x(2x-3y)+4y(2x-3y)$

$(2x-3y)(3x+4y)$

e.) $27p^{10}-45p^9-252p^8$

$9p^8(3p^2-5p-28)$ $\quad -12\cdot 7=-84$, $-12+7=-5$

$\boxed{9p^8(p-4)(3p+7)}$

$3p^2-12p+7p-28$

$(3p^2-12p)+(7p-28)$

$3p(p-4)+7(p-4)$

$(p-4)(3p+7)$

Question #5 continued

f) $x^2-17x+66$

$-6 \cdot -11 = 66$
$-6 + -11 = -17$

$x^2-6x-11x+66$

$(x^2-6x)-(11x-66)$

$x(x-6)-11(x-6)$

$\boxed{(x-6)(x-11)}$

g) $y^2-4yk-12k^2$

$-6 \cdot 2 = -12$
$-6+2 = -4$

$\boxed{(y-6k)(y+2k)}$

h) $4x^5+12x^4-40x^3$

$4x^3(x^2+3x-10)$

$-2 \cdot 5 = -10$
$-2+5 = 3$

$\boxed{4x^3(x-2)(x+5)}$

i) $20x^2+22x+6$

$2(10x^2+11x+3)$

$5 \cdot 6 = 30$
$5+6 \quad 11$

$\boxed{2(2x+1)(5x+3)}$

$10x^2+5x+6x+3$

$(10x^2+5x)+(6x+3)$

$5x(2x+1)+3(2x+1)$

$(2x+1)(5x+3)$

j) $9p^2-18p+8$

$-12 \cdot -6 = 72$
$-12 + -6 = -18$

$9p^2-12p-6p+8$

$(9p^2-12p)-(6p-8)$

$3p(3p-4)-2(3p-4)$

$\boxed{(3p-4)(3p-2)}$

k) $24x^2-42x+9$

$3(8x^2-14x+3)$

$-12 \cdot -2 = 24$
$-12+-2 = -14$

$\boxed{3(2x-3)(4x-1)}$

$8x^2-12x-2x+3$

$(8x^2-12x)-(2x-3)$

$4x(2x-3)-1(2x-3)$

$(2x-3)(4x-1)$

l) $6x^2-5xy-y^2$

$-6 \cdot 1 = -6$
$-6+1 = -5$

$6x^2-6xy+xy-y^2$

$(6x^2-6xy)+(xy-y^2)$

$6x(x-y)+y(x-y)$

$\boxed{(x-y)(6x+y)}$

m) $24x^4+10x^3-4x^2$

$2x^2(12x^2+5x-2)$

$-3 \cdot 8 = -24$
$-3+8 = 5$

$\boxed{2x^2(4x-1)(3x+2)}$

$12x^2-3x+8x-2$

$(12x^2-3x)+(8x-2)$

$3x(4x-1)+2(4x-1)$

$(4x-1)(3x+2)$

n) $2w^3-2w^2+3w-3$

$(2w^3-2w^2)+(3w-3)$

$2w^2(w-1)+3(w-1)$

$\boxed{(w-1)(2w^2+3)}$

o) $15x^2-14x-8$

$-20 \cdot 6 = -120$
$-20+6 = -14$

$15x^2-20x+6x-8$

$(15x^2-20x)+(6x-8)$

$5x(3x-4)+2(3x-4)$

$\boxed{(3x-4)(5x+2)}$

p) $-3x^3+27x$

$-3x(x^2-9)$

$\boxed{-3x(x+3)(x-3)}$

q) $9x^2-12xy+4y^2$

$(3x)^2 \qquad (2y)^2$

$2(3x)(2y) = 12xy$

$\boxed{(3x-2y)^2}$

Perfect Square Trinomial

Question #5 continued.

r.) $x^2-12x-28$ $-14\cdot 2=-28$, $-14+2=-12$

$\boxed{(x-14)(x+2)}$

s) $y^3+5y^2-4y-20$

$(y^3+5y^2)-(4y+20)$

$y^2(y+5)-4(y+5)$

$(y+5)(y^2-4)$

$\boxed{(y+5)(y+2)(y-2)}$

t.) $24x^2-46x+10$ $-20\cdot -3=60$, $-20+-3=-23$

$2(12x^2-23x+5)$

$12x^2-20x-3x+5$

$(12x^2-20x)-(3x-5)$

$4x(3x-5)-1(3x-5)$

$(3x-5)(4x-1)$

$\boxed{2(3x-5)(4x-1)}$

u.) $27x^3-64y^3$

$(3x)^3-(4y)^3$ difference of cubes

$\boxed{(3x-4y)(9x^2+12xy+16y^2)}$

v.) $10x^2+19x+6$ $15\cdot 4=60$, $15+4=19$

$10x^2+15x+4x+6$

$(10x^2+15x)+(4x+6)$

$5x(2x+3)+2(2x+3)$

$\boxed{(2x+3)(5x+2)}$

w.) $6x^2-7xy-5y^2$ $-10\cdot 3=-30$, $-10+3=-7$

$6x^2-10xy+3xy-5y^2$

$(6x^2-10xy)+(3xy-5y^2)$

$2x(3x-5y)+y(3x-5y)$

$\boxed{(3x-5y)(2x+y)}$

x.) $64x^2-16y^2$

$16(4x^2-y^2)$ ← difference of squares

$(2x)^2-(y)^2$

$\boxed{16(2x+y)(2x-y)}$

y.) $x^2-14x+49$ $-7\cdot -7=49$, $-7=-7-14$

$(x-7)(x-7)$

$\boxed{(x-7)^2}$

z.) $9x^2+48xy+64y^2$ $2(3x)(8y)=48xy$

$(3x)^2$ $(8y)^2$

$\boxed{(3x+8y)^2}$

aa.) $2x^3y-32xy$

$2y(x^2-16)$

$\boxed{2y(x+4)(x-4)}$

Question #6

a.) $x^2-x-20=0$

$(x-5)(x+4)=0$

$x-5=0$ OR $x+4=0$

$\boxed{x=5 \text{ OR } x=-4}$

b.) $x^2-4x=12$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x-6=0$ OR $x+2=0$

$\boxed{x=6 \text{ OR } x=-2}$

Question #6 continued

c.) $6x^2 - 7x + 2 = 0$

$(3x-1)(x-2) = 0$

$3x - 1 = 0$ or $x - 2 = 0$

$\frac{3x}{3} = \frac{1}{3}$

$x = 1/3$ or $x = 2$

d.) $6x^2 = 4 + 5x$

$6x^2 - 5x - 4 = 0$

$(2x+1)(3x-4) = 0$

$2x + 1 = 0$ or $3x - 4 = 0$

$\frac{2x}{2} = \frac{-1}{2}$ $\quad$ $\frac{3x}{3} = \frac{4}{3}$

$x = -1/2$ OR $x = 4/3$

e.) $x^2 = 121$

$x^2 - 121 = 0$

$(x+11)(x-11) = 0$

$x + 11 = 0$ or $x - 11 = 0$

$x = -11$ OR $x = 11$

f.) $(x-6)(2x^2 - 5x - 3) = 0$

$(x-6)(2x+1)(x-3) = 0$

$x - 6 = 0$ or $2x + 1 = 0$ or $x - 3 = 0$

$x = 6$ or $x = -1/2$ or $x = 3$

g.) $4x(2x+3) = 36$

$8x^2 + 12x = 36$

$8x^2 + 12x - 36 = 0$

$4(2x^2 + 3x - 9) = 0$

$4(2x-3)(x+3) = 0$

$4 = 0$ or $2x - 3 = 0$ or $x + 3 = 0$

X $\quad$ $x = 3/2$ or $x = -3$

Question #7

1st: x

2nd: $x+1$

$x(x+1) = 4(x+x+1) - 4$

$x^2 + x = 4(2x+1) - 4$

$x^2 + x = 8x + 4 - 4$

$x^2 + x = 8x$

$x^2 + x - 8x = 0$

$x^2 - 7x = 0$

$x(x-7) = 0$

$x = 0$ or $x = 7$

1st: 0 $\quad$ 1st 7

2nd: 1 $\quad$ 2nd 8

The integers are 0, 1 or 7, 8

Question #8

length $= l$

width $= w$

$w = l - 3$ $\quad$ height $= 2$ ft

$V = lwh$

$80 = l(l-3)(2)$

$80 = 2l^2 - 6l$

$0 = 2l^2 - 6l - 80$

$0 = 2(l^2 - 3l - 40)$

$0 = 2(l-8)(l+5)$

$2 = 0$ or $l - 8 = 0$ or $l + 5 = 0$

X $\quad$ $l = 8$ or $l = -5$

↑ length cannot be negative

$l = 8$

$w = 8 - 3 = 5$

The length is 8 ft and the width is 5 ft

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Question #9

1st: x
2nd: $x+2$
3rd: $x+4$

$$x + x+2 + x+4 = (x+2)(x+4) - 42$$
$$3x+6 = x^2+6x+8-42$$
$$3x+6 = x^2+6x-34$$
$$0 = x^2+6x-34-3x-6$$
$$0 = x^2+3x-40$$
$$0 = (x+8)(x-5)$$

$x+8=0$ or $x-5=0$
$x=-8$ or $x=5$
↑ even ↑ odd.

The integers are 5, 7, 9.

Question #10

length $= l$
width $= w$

$l = 3+w$

$A = lw$
$28 = (3+w)w$
$28 = 3w+w^2$
$0 = w^2+3w-28$
$0 = (w+7)(w-4)$

$w+7=0$ or $w-4=0$
$w=-7$ or $w=4$
↑
width cannot be negative

$w=4 \rightarrow l = 3+4 = 7$

The width is 4 in and the length is 7 in.

Question #11

only a, f, and g

b, c, d, e, h, i are not on Exam #2

a.) $\frac{x^5y^2}{x^2y^4} \cdot \frac{y^6}{x^5x^2}$

$\frac{y^2}{x^2y^4} \cdot \frac{y^6y^2}{x^2}$

$\frac{y^2}{x^2} \cdot \frac{y^2}{x^2} = \boxed{\frac{y^4}{x^4}}$

f.) $\frac{2y^2+y-6}{2y^2-9y+9} \cdot \frac{y^2-2y-3}{y^2-1}$

$\frac{(2y-3)(y+2)\ (y-3)(y+1)}{(2y-3)(y-3)\ (y+1)(y-1)}$

$\frac{y+2}{y-1}$

g) $\frac{3t^2-t}{6t^2+15t} \div \frac{6t^2+t-1}{2t^2-5t-25}$

$\frac{(3t^2-t)(2t^2-5t-25)}{(6t^2+15t)(6t^2+t-1)}$

$\frac{t(3t-1)(2t+5)(t-5)}{3t(2t+5)(3t-1)(2t+1)}$

$\frac{t-5}{3(2t+1)}$

Question #1 continued

e.) $5 < 1 - 6m < 12$

$5 - 1 < 1 - 6m - 1 < 12 - 1$

$4 < -6m < 11$

$\frac{4}{-6} > \frac{-6m}{-6} > \frac{11}{-6}$

$-\frac{2}{3} > m > -\frac{11}{6}$

$-\frac{11}{6} < m < -\frac{2}{3}$

$\left(-\frac{11}{6}, -\frac{2}{3}\right)$

f.) $-7 \leq 3x - 4 \leq 8$

$-7 + 4 \leq 3x - 4 + 4 \leq 8 + 4$

$-3 \leq 3x \leq 12$

$\frac{-3}{3} \leq \frac{3x}{3} \leq \frac{12}{3}$

$-1 \leq x \leq 4$

$[-1, 4]$

g.) $2 < 6 + \frac{3}{4}x \leq 12$ LCD = 4

$4(2) < 4(6) + 4\left(\frac{3}{4}x\right) \leq 4(12)$

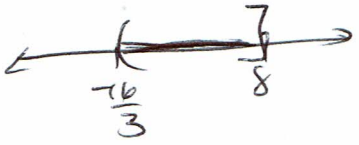
$8 < 24 + 3x \leq 48$

$8 - 24 < 24 + 3x - 24 \leq 48 - 24$

$-16 < 3x \leq 24$

$\frac{-16}{3} < \frac{3x}{3} \leq \frac{24}{3}$

$-\frac{16}{3} < x \leq 8$



$\left(-\frac{16}{3}, 8\right]$

h.) $-12 \leq \frac{3}{7}x + 2 < -4$ LCD = 7

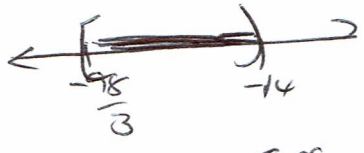
$7(-12) \leq 7\left(\frac{3}{7}x\right) + 7(2) < 7(-4)$

$-84 \leq 3x + 14 < -28$

$-84 - 14 \leq 3x + 14 - 14 < -28 - 14$

$\frac{-98}{3} \leq \frac{3x}{3} < \frac{-42}{3}$

$-\frac{98}{3} \leq x < -14$



$\left[-\frac{98}{3}, -14\right)$