

Math 71 Final Exam Review Sheet

Please Note: The review sheet is designed for you to have a guide as to what to study. The problems on the exam are not limited to the type of problems on this sheet. Any type of problem from the assigned homework problems are possible exam questions. Please attempt other practice problems other than those presented on this sheet in order to be completely prepared for the exam.

1. Solve the following equations.

a. $\frac{x}{2} - 5 = -12 - \frac{2x}{3}$

b. $6 - 4|x + 3| = -2$

c. $x^2 = 6x - 13$

d. $x^4 - 7x^2 + 12 = 0$

e. $x^2 + 6x = 7$

f. $4x^2 + 8x + 3 = 0$

g. $x^4 - 5x^2 + 4 = 0$

h. $x^2 = -12x + 13$

i. $2x^2 + 12x = -5$

j. $(x + 3)(x + 2) = 15$

k. $6y^2 + 2y + 1 = 0$

l. $\sqrt{3x + 1} = 6$

m. $\sqrt[3]{x + 5} = 2$

n. $3 + \sqrt{x - 6} = \sqrt{x + 9}$

o. $\frac{x - 2}{2x} + 1 = \frac{x + 1}{x}$

p. $\frac{x + 2}{x + 10} = \frac{x - 3}{x + 4}$

q. $\frac{8}{x^2 - 9} + \frac{4}{x + 3} = \frac{2}{x - 3}$

r. $\frac{1}{x - 2} + \frac{1}{x + 2} = \frac{4}{x^2 - 4}$

s. $\frac{2x - 1}{x^2 + 2x - 8} + \frac{2}{x + 4} = \frac{1}{x - 2}$

2. Graph each of the following quadratic functions. Determine the vertex, axis of symmetry, and intercepts, if they exist.

a. $f(x) = x^2 - 6x + 9$

b. $f(x) = 2x^2 - x + 2$

c. $f(x) = 2x^2 + 5x + 3$

3. Solve the following inequalities.

a. $\frac{1}{2} - x > \frac{x}{3} + \frac{1}{4}$

b. $|3x - 7| \geq -5$

c. $|4x - 3| \leq 5$

d. $\frac{x-4}{x+2} \leq 0$

e. $5x + 1 \geq 3x - 9$

f. $4x - (6x + 1) \leq 8x + 2(x - 3)$

g. $2(x - 5) + 3x < 4(x - 6) + 1$

h. $5 < 1 - 6m < 12$

i. $6 - 5x > 1 - 3x$ and $4x - 3 > x - 9$

j. $3 \leq 4x - 3 < 19$

k. $3x + 2 \leq 5$ or $5x - 7 \geq 8$

l. $5(x - 2) > 15$ and $\frac{x-6}{4} \leq -2$

m. $\frac{x^2 - 4x + 3}{x + 4} \geq 0$

n. $x^2 + 4x < 12$

o. $\frac{x+4}{x-2} \leq 1$

$$p. \frac{x+1}{x-3} \geq 0$$

4. Simplify the following complex expressions.

$$a. (5-3i) - (-3+2i)$$

$$b. (4-3i)(2+5i)$$

$$c. \frac{2-i}{3+4i}$$

$$d. i^{3513}$$

5. Given $y = 2x^2 + 5x - 3$.

a. Rewrite the equation in standard graphing form of a parabola.

b. Identify the vertex.

c. Identify the axis of symmetry.

6. Solve the following system of equations.

$$a. \begin{cases} x-2y=16 \\ y+3=3x \end{cases}$$

$$b. \begin{cases} x-3y=-6 \\ 3x-9y=9 \end{cases}$$

$$c. \begin{cases} 3x-5y=11 \\ 2x-6y=2 \end{cases}$$

$$d. \begin{cases} 3x+2y-z=4 \\ 3x-2y+z=5 \\ 4x-5y-z=-1 \end{cases}$$

7. Simplify the following radical expressions.

$$a. \sqrt{18} - \sqrt{50} + \sqrt{12} - \sqrt{75}$$

$$b. \frac{5}{3-\sqrt{2}}$$

$$c. 5\sqrt{12} + 16\sqrt{27}$$

$$d. \sqrt{5a} + 2\sqrt{45a^3}$$

e. $\sqrt{10} \cdot \sqrt{5}$

f. $\sqrt{5a^7} \cdot \sqrt{15a^3}$

g. $\frac{\sqrt{40xy^3}}{\sqrt{8x}}$

h. $\frac{4\sqrt{5}}{3\sqrt{2}}$

i. $\frac{3-\sqrt{2}}{5+\sqrt{6}}$

8. Simplify the following expressions.

a. $\left(\frac{1}{81} - \frac{1}{x^2}\right) \div (x-9)$

b. $\frac{a^2-9}{3a-6} \cdot \frac{a^2-4}{a^2-a-6}$

c. $\left(\frac{-2x^4y^{-4}}{3x^{-3}y^{-2}}\right)^{-3}$

d. $\frac{x^2-16}{x^2-10x+25} \div \frac{3x-12}{x^2-3x-10}$

e. $\frac{4}{x+1} + \frac{x+2}{x^2-1} + \frac{3}{x-1}$

f. $\frac{\frac{4}{x^2-1} - \frac{3}{x+1}}{\frac{x^2-1}{5} - \frac{x+1}{2}}$

g. $\frac{\frac{x^{-1}+y^{-1}}{x^2-y^2}}{xy}$

9. Factor completely.

a. $2w^3 - 2w^2 + 3w - 3$

b. $15x^2 - 14x - 8$

c. $-3x^3 + 27x$

d. $9x^2 - 12xy + 4y^2$

e. $a^8 - b^8$

f. $8m^3 + m^6 - 20$

g. $10x^2 + 19x + 6$

h. $6x^2 - 7xy - 5y^2$

i. $(x-6)^2 - y^2$

j. $64x^2 - 16y^2$

k. $x^2 - 14x + 49$

l. $9x^2 + 48xy + 64y^2$

m. $2x^3y - 32xy$

n. $x^2 - 12x + 36 - y^2$

10. Perform the indicated operation.

a. $(-9xy^2 - xy + 6x^2y) + (-5x^2y - xy + 4xy^2) + (12x^2y - 3xy^2 + 6xy)$

b. $(4x-3)(2x+7)$

c. $(x^2 - 2x + 1)(x^2 + x + 2)$

d. $(x^3 - 2x^2 + 2x - 5) \div (x+1)$

11. Given $f(x) = 3x^2 + 2$, $g(x) = 4x - 3$ and $h(x) = |3x - 2|$. Find the following:

a. $f(4)$

b. $h(-4)$

c. $f(x+2)$

d. $(g \circ h)(x)$

e. $g^{-1}(x)$

12. Find the 100th term of $a_n = \frac{3n+2}{n^2+4}$

13. Multiply using the binomial theorem.

a. $(x+y)^5$

b. $(2x - 3y)^4$

14. Given the following equations of a circle. For each, identify the center and the radius. Then, sketch the graph.

a. $(x + 4)^2 + (y - 3)^2 = 10$

b. $x^2 + y^2 - 10x - 6y - 30 = 0$

15. Given the following equations of an ellipse. For each, identify the center, the vertices, and the major axis. Then, sketch the graph.

a. $12(x - 1)^2 + 3(y + 4)^2 = 48$

b. $9(x - 1)^2 + 4(y + 3)^2 = 36$

16. Given the following equations of a hyperbola. For each, identify the center and the vertices. Then, sketch the graph.

a. $\frac{x^2}{4} - \frac{y^2}{25} = 1$

b. $\frac{y^2}{49} - \frac{x^2}{36} = 1$

17. Given the following equations of a parabola. For each, identify the vertex, the axis of symmetry, the x -intercept(s), and the y -intercept(s). Then, sketch the graph.

a. $y = 2(x - 3)^2 + 1$

b. $x = (y - 4)^2 + 1$

c. $x = y^2 - 6y + 8$

18. Solve the following system of equations.

a.
$$\begin{cases} y = x^2 + 1 \\ y = 4x + 1 \end{cases}$$

b.
$$\begin{cases} 2x^2 + y^2 = 18 \\ xy = 4 \end{cases}$$

$$c. \begin{cases} 7x^2 - 3y^2 + 5 = 0 \\ 3x^2 + 5y^2 = 12 \end{cases}$$

19. Graph the following functions.

a. $f(x) = 2^{x-1} + 4$

b. $f(x) = \log_2(x+3) - 4$

20. Evaluate the following.

a. $\log_5 25$

b. $\log_2 \frac{1}{8}$

c. $\log_7 \sqrt{7}$

d. $\log_5 5$

e. $\log_6 1$

f. $\log_4 4^6$

g. $8^{\log_8 19}$

h. $\log_2 64$

i. $\log_{49} 7$

21. Evaluate the following logarithms by rounding to the nearest thousandth.

a. $\log 52$

b. $\ln 28$

c. $\log_5 12$

d. $\log_7 31$

22. Solve for x .

a. $2^{2x+1} = 4$

b. $3^{x^3} = 9^x$

c. $5^{2x-1} = 125$

d. $4^x = \frac{1}{\sqrt{2}}$

e. $3^x = 14$

$$f. 3^{1-2x} = 4^x$$

$$g. e^{5x} = 7$$

$$h. 9e^x = 99$$

$$i. e^{0.08x} = 4$$

$$j. \log_2(x^2) - \log_2(x-2) = 3$$

$$k. \log_2(x-3) + \log_2(x+4) = 3$$

$$l. x = \log_5 625$$

$$m. \log_8 x = -2$$

$$n. \log_x 4 = \frac{1}{3}$$

$$o. \log_4(x^2 - 9) - \log_4(x + 3) = 3$$

$$p. \log(3x - 5) - \log 5x = 2$$

23. Express the following as a sum and/or difference of logarithms.

$$a. \log_3 \frac{x^4 y^3}{z^2 w^3}$$

$$b. \log \sqrt{\frac{x^2}{y^3}}$$

24. Express the following sum and difference of logarithms as a single logarithm.

$$a. 2\log_3 x + 4\log_3 y - 6\log_3 z$$

$$b. 6\log_4 x - 2\log_4 y + 3\log_4 z$$

25. Find the distance between the given pairs of points.

$$a. (-2, -6) \text{ and } (3, -4)$$

$$b. (3\sqrt{3}, \sqrt{5}) \text{ and } (-\sqrt{3}, 4\sqrt{5})$$

26. Find the midpoint of the line segment with the given endpoints.

- a. $(10,4)$ and $(2,6)$
- b. $(-2,-1)$ and $(-8,6)$

27. Find the slope of the line passing through the following pairs of points.

- a. $(3,4)$ and $(7,-2)$
- b. $(-1,-4)$ and $(5,3)$
- c. $(4,7)$ and $(8,7)$
- d. $(2,4)$ and $(2,6)$

28. Find the slope, x -intercept and y -intercept of the following linear equations. Then, graph the equation.

- a. $y = \frac{2}{3}x - \frac{3}{4}$
- b. $5x - 4y = 20$
- c. $2x + y = 6$
- d. $x = 5$
- e. $y = 2$

29. For each pair of equations, give the slopes of the lines and then determine whether the two lines are parallel, perpendicular, or neither parallel nor perpendicular.

- a. $-4x + 3y = 4$
 $-8x + 6y = 0$
- b. $5x - 3y = -2$
 $3x - 5y = -8$
- c. $3x - 5y = -1$
 $5x + 3y = 2$

30. Graph the following inequalities.

- a. $2x - 3y \geq 6$
- b. $3x + 5y \leq 10$
- c. $2x - y \leq 4$
- d. $3x + 2y > -6$

31. Given the following functions.

$$f(x) = 3x^2 - 4x + 2$$

$$g(x) = \frac{5}{x-4}$$

$$h(x) = -4x^2 + 2x + 8$$

Evaluate:

a. $f(2)$

b. $g(1)$

c. $h(-3)$

32. Find the domains of the following functions.

a. $f(x) = \frac{3}{2x-5}$

b. $f(x) = 2x + 1$

c. $f(x) = \frac{7x}{5-x}$

33. A rectangular soccer field is twice as long as it is wide. If the perimeter of the soccer field is 300 yards, what are its dimensions?

34. A rectangular pool is 6 meters less than twice the width. If the pool's perimeter is 126 meters, what are its dimensions?

35. The speed of the current in Willow Creek is 3 mph. Bill's kayak can travel 4 mi upstream in the same time it takes to travel 10 mi downstream. What is the speed of Bill's kayak in still water?

36. Alvin paddled for 4 hr with a 6-km/h current to reach a campsite. The return trip against the same current took 10 hr. Find the speed of Alvin's canoe in still water.

37. Casella's Catering is planning a wedding reception. The bride and groom would like to serve a nut mixture containing 25% peanuts. Casella has available mixtures that are either 40% or 10% peanuts. How much of each type should be mixed to get a 10-lb mixture that is 25% peanuts?

38. Two Quarter Pounders and three Whoppers with cheese provide 2607 calories. One of each provides 9 calories in excess of what is allowed on a 1000 calorie-a-day diet. Find the calories in each item.

39. A grocer needs to mix tea worth \$6.00 per pound with tea worth \$8.00 per pound to obtain 144 pounds of a tea mixture worth \$7.50 per pound. How many pounds of each kind of tea must be used?
40. The sum of three numbers is 16. The sum of twice the first number, 3 times the second number, and 4 times the third number is 46. The difference between 5 times the first number and the second number is 31. Find the three numbers.
41. The foot of an extension ladder is 10 ft from a wall. The ladder is 2 ft longer than the height that it reaches on the wall. How far up the wall does the ladder reach?
42. A rectangular garden is 60 ft by 80 ft. Part of the garden is removed in order to install a walkway of uniform width around it. The area of the new lawn is one-half the area of the old garden. How wide is the walkway?
43. In still water, a boat averages 8 miles per hour. It takes the same amount of time to travel 15 miles downstream, with the current, as 9 miles upstream, against the current. What is the rate of the water's current?
44. A pool can be filled by one pipe in 3 hours and by a second pipe in 6 hours. How long will it take using both pipes to fill the pool?
45. Working together, Lou and Bud can paint a room in 6 hours. Working alone, it takes Lou 5 hours longer than Bud to do the job. How long would it take Bud to paint the room alone?
46. A slow-pitch softball diamond is actually a square 65 feet on a side. How far is it from home plate to second base?
47. Find the amount that results from each investment.
- \$50 invested at 6% compounded monthly after a period of 3 years.
 - \$700 invested at 6% compounded daily after a period of 2 years.
 - \$100 invested at 12% compounded continuously after a period of $3\frac{3}{4}$.
48. How many years will it take for an initial investment of \$10,000 to grow to \$25,000? Assume a rate of interest of 6% compounded continuously.
49. The number N of bacteria present in a culture at a time t (in hours) obeys the function $N(t) = 1000e^{0.01t}$. After how many hours will the population equal 1500? 2000?
50. The population of a southern city follows the exponential law. If the population doubled in size over an 18-month period and the current population is 10,000, what will the population be 2 years from now? (*Hint: Find k first*)
51. The half-life of radium is 1690 years. If 10 grams is present now, how much will be present in 50 years? (*Hint: Find k first*)

52. A piece of charcoal is found to contain 30% of the carbon 14 that it originally had. When did the tree from which the charcoal came die? Use 5600 years as the half-life of carbon 14.
53. An architect is designing a rectangular family room with a perimeter of 56 feet. What dimensions will yield the maximum area? What is the maximum area?
54. Economite Plastics plans to produce a one-compartment vertical file by bending the long side of an 8-inch by 14-inch sheet of plastic along two lines to form a U shape. How tall should the file be in order to maximize the volume that the file can hold?